

# Fitness, chance, and myths: an objective view on soccer results

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## Abstract

We analyze the time series of soccer matches in a model-free way using data for the German soccer league (Bundesliga). We argue that the goal difference is a better measure for the overall fitness of a team than the number of points. It is shown that the time evolution of the table during a season can be interpreted as a random walk with an underlying constant drift. Variations of the overall fitness mainly occur during the summer break but not during a season. The fitness correlation shows a long-time decay on the scale of a quarter century. Some typical soccer myths are analyzed in detail. It is shown that losing but no winning streaks exist. For this analysis ideas from multidimensional NMR experiments have been borrowed. Furthermore, beyond the general home advantage there is no statistically relevant indication of a team-specific home fitness. Based on these insights a framework for a statistical characterization of the results of a soccer league is introduced and some general consequences for the prediction of soccer results are formulated.

PACS numbers: 89.20.-a, 02.50.-r

## I. INTRODUCTION

In recent years physicists have started to investigate time series, resulting from successive matches in sports leagues. In this context several basic questions can be asked. Is the champion always the best team? [1, 2, 3] How many matches have to be played in a league so that (nearly) always the best team becomes the champion? [1, 2] Does the distribution of goals follow a Poisson distribution and what are possible interpretations of the observed deviations? [4, 5]. In those studies it has been attempted to have a simplified view on complex processes such as soccer matches in order to extract some basic features like, e.g., scaling laws. Some empirical observations such as fat tails in the goal distributions can be related to other fields such as finance markets [6] and have been described, e.g., by the Zipf-Mandelbrot law [7]. Actually, also in more general context the analysis of sports events, e.g. under the aspect of extreme value statistics, has successfully entered the domain of physicists activities [8].

A more specific view has been attempted in detailed studies of the course of a soccer season. In one type of models; see e.g. Refs. [9, 10, 11, 12], one introduces different parameters to characterize a team (e.g. offensive fitness) which can be obtained via Monte-Carlo techniques. These parameters are then estimated based on a Poisson assumption about the number of goals of both teams. Within these models, which were mainly applied to the English Premier league, some temporal weighting factors were included to take into account possible time variations of the different team parameters. These models are aimed to make predictions for the goals in individual matches. In [12] it is reported that based on a complex fitting procedure the time scale of memory loss with respect to the different variables is as short as 100 days.

A second type of model assumes just one fitness parameter for each team and the outcome (home win, draw, away win) is then predicted after comparing the difference of the team fitness parameters with some fixed parameters [13]. The model parameters are then estimated based on the results of the whole season. Here, no temporal evolution of the team parameter is involved. This very simple model has been used in [14] to check whether the outcome of one match influences the outcome of the successive match. Of course, this type of results is only relevant if the used model indeed reflects the key ingredients of the real soccer events in a correct way. It has been also attempted to analyse individual soccer

matches on a very detailed level, e.g., to estimate the effect of tactical changes [15]

The approach, taken in this work, is somewhat different. Before devising appropriate models, which will be done in subsequent work, we first attempt to use a model-free approach to learn about some of the underlying statistical features of German soccer (1. Bundesliga). However, the methods are general enough so that they can be easily adapted to different soccer leagues or even different types of sports. The analysis is exclusively based on the knowledge of the final results of the individual matches. Since much of the earlier work in this field originates from groups with a statistics or economy background, there is some room for the application of complimentary concepts, more common in the physics community. Examples are finite-size scaling, the analysis of 2-time correlation functions or the use of more complex correlation functions to unravel the properties of subensembles, as used, e.g., in previous 4D NMR experiments [16, 17, 18].

Four key goals are followed in this work. First, we ask about appropriate observables to characterize the overall fitness of a team. Second, using this observable we analyze the temporal evolution of the fitness on different time scales. Third, we quantify statistical and systematic features for the interpretation of a league table and derive some general properties of prediction procedures. Forth, we clarify the validity of some soccer myths which are often used in the typical soccer language, including serious newspapers, but never have been fully checked about their objective validity. Does something like a winning or losing streak exist? Do some teams have a specific home fitness during one season?

The paper is organized as follows. In Sect.II we briefly outline our data basis. The discussion of the different possible measures of the overall fitness is found in Sect.III. In the next step the temporal evolution of the fitness is analyzed (Sect.IV). In Sect.V it is shown how the systematic differences in the team fitness can be separated from the statistical effects of soccer matches and how a general statistical characterization can be performed. In Sect.VI we present a detailed discussion of some soccer myths. Finally, in Sect.VII we end with a discussion and a summary. In two appendices more detailed results about a few aspects of our analysis are presented.

## II. DATA BASIS

We have taken the results of the German Bundesliga from <http://www.bundesliga-statistik.de>. For technical reasons we have excluded the seasons 1963/64, 1964/65 and 1991/92 because these were the seasons where the league contained more or less than 18 teams. Every team plays against any other team twice the season, once at home and once away. If not mentioned otherwise we have used the results starting from the season 1987/88. The reason is that in earlier years the number of goals per season was somewhat larger, resulting in slightly different statistical properties.

## III. USING GOALS OR POINTS TO MEASURE THE TEAM FITNESS?

### A. General problem

Naturally, a strict characterization of the team fitness is not possible because human behavior is involved in a complex manner. A soccer team tries to win as many matches as possible during a season. Of course, teams with a better fitness will be more successful in this endeavor. As a consequence the number of points  $P$  or the goal difference  $\Delta G$  can be regarded as a measure for the fitness. In what follows all observables are defined as the average value per match.

In Sect.IV it is shown that apart from fluctuations the team fitness remains constant during a season. Thus, in a hypothetical season where teams play infinitely often against each other and thus statistical effects are averaged out the values of  $P$  indeed allow a strict sorting of the quality of the teams. Thus,  $P$  is a well-defined fitness measure for the team fitness during a season. Naturally, the same holds for  $\Delta G$  if the final ranking would be related to the goal difference. Since in reality the champion is determined from the number of points one might tend to favor  $P$  to characterize the team fitness. In any event, one would expect that the rankings with respect to  $\Delta G$  or  $P$  are identical in this hypothetical limit.

Evidently, in a match the number of goals scored or conceded by a team is governed by many unforeseen effects. This is one of the reasons why soccer is so popular. As a consequence, the empirical values of  $P$  or  $\Delta G$  obtained, e.g., after a full season will deviate from the limiting values due to the residual fluctuations. This suggests a relevant criterion to distinguish between different observables. Which observable displays a minimum sensitivity

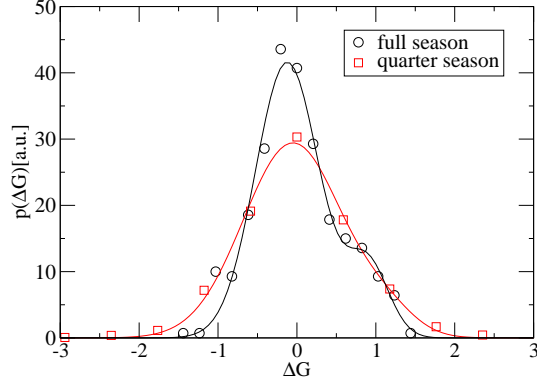


Figure 1: The distribution of  $\Delta G$  after one quarter of the season and after a full season. Included is a fit with two Gaussian functions for both distributions. For the full-season distribution the intensity ratio of both Gaussian curves is approx. 1:6. The correlation coefficient for the latter is 0.985.

on statistical effects? As will be shown below, this criterion favors the use of  $\Delta G$ .

## B. Distribution of $\Delta G$

In Fig.1 we display the distribution of  $\Delta G$  after one quarter of a season (thereby averaging over all quarters) and at the end of the season. The first case corresponds to  $N = 9$  (first and third quarter) or 8 (second and fourth quarter), the second case to  $N = 34$ . Here  $N$  denotes the number of subsequent matches, included in the determination of  $\Delta G$ .

Both distributions can be described as a Gaussian plus an additional wing at large  $\Delta G$ . Fitting each curve by a sum of two Gaussians, the amplitude ratio for the full-season distribution implies that there are on average 2-3 teams with an exceptional good fitness.

Note that the distribution of  $\Delta G$  is significantly narrower for larger  $N$  and also for  $N = 34$  one expects some finite statistical contribution to the width of the distribution. Qualitatively, this reflects the statistical nature of individual soccer matches. Naturally, the statistical contribution becomes less relevant when averaging over more matches. This averaging effect will be quantified in Sect.V.

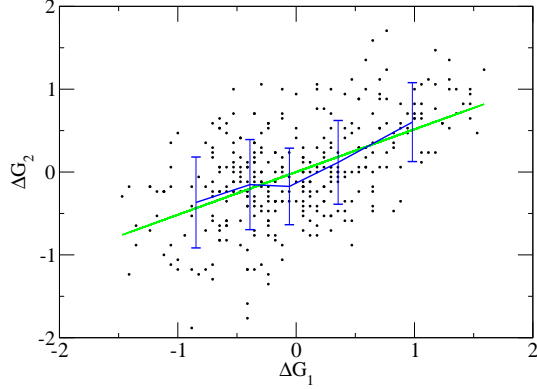


Figure 2: The correlation of  $\Delta G$  for the first and the second half of the season. Included are the respective averages together with the standard deviation which on average is 0.51. Furthermore an overall regression line is included which has a slope of 0.53.

### C. Correlation analysis

A natural question to ask is whether the distribution for  $N = 34$  can be explained under the assumption that all teams have an identical fitness. If this is the case the outcome of each match would be purely statistical and no correlation between the goal differences of a team in successive matches could be found. To check this possibility in a simple manner we correlate the value of  $\Delta G$ , obtained in the first half of the season ( $\Delta G_1$ ), with the value of the second half of the same team ( $\Delta G_2$ ). The results, collected for all years and all teams (per year) are shown in Fig.2. One observes a significant correlation. Thus, not surprisingly, there is indeed a variance of the fitness of different teams.

For a quantification of the correlation one can use the Pearson correlation coefficient

$$c_P(M_1, M_2) = \frac{\langle (M_1 - \langle M_1 \rangle)(M_2 - \langle M_2 \rangle) \rangle}{\sigma_{M,1}\sigma_{M,2}} \quad (1)$$

to correlate two distributions  $M_1$  and  $M_2$ . For the present problem it yields  $0.55 \pm 0.03$ . The error bar has been determined by calculating  $c_P(M_1, M_2)$  individually for every year and then averaging over all years. This procedure is also applied in most of the subsequent analysis and allows a straightforward estimation of the statistical uncertainty. The average value  $\langle \Delta G_2 \rangle$  can be interpreted as the best estimation of the fitness, based on knowledge of  $\Delta G_1$ . Note that the variance of the distribution of  $\Delta G_2$  for every  $\Delta G_1$  is basically independent of  $\Delta G_1$  and is given by 0.51.

There is a simple but on first view astonishing observation. It turns out that a team

with a positive  $\Delta G$  in the first half will on average also acquire a positive  $\Delta G$  in the second half, but with a smaller average value. This is reflected by the slope of the regression line smaller than unity. This observation is a manifestation of the regression toward the mean [19], which, however, is not always taken into account [3]. Qualitatively, this effect can be rationalized by the observation that a team with a better-than-average value of  $\Delta G$  very likely has a higher fitness but, at the same time, on average also had some good luck. This statistical bias is, of course, not repeated during the second half of the season. For a stationary process  $\Delta G$  has the same statistical properties in the first and the second half. Then the slope of the regression line is identical to the correlation coefficient (here: 0.53 vs. 0.55).

In a next step we have taken the observable  $p(\Delta G = 2)$  which describes the probability that a team wins a match with a goal difference of exactly two. Of course, this is also a measure of the fitness of the team but intuitively one would expect a major intrinsic statistical variance which should render this observable unsuited to reflect the team fitness for the real situation of a finite season. One obtains a correlation coefficient of 0.19. In agreement with intuition one indeed sees that observables which are strongly hampered by statistical effects display a lower correlation coefficient. Stated differently, the value of  $c_p(M_1, M_2)$  can be taken as a criterion how well the observable  $M$  reflects the fitness of a team. This statement is further corroborated in Appendix I on the basis of a simple model calculation. In particular it is shown that this statement holds whether or not the team fitness changes during a season.

We have repeated the analysis for the value of  $P$ , applying the present rule (3 points for a win, 1 point for a draw and 0 for a loss) to all years. The results, however, are basically identical if using the 2-point rule. Here we obtain  $0.49 \pm 0.03$  which is smaller than the value obtained for  $\Delta G$ . One might argue that both values can still agree within statistical errors. However, since the variation from season to season is very similar for both correlation factors the difference is indeed significant. A detailed statistical analysis yields  $c_P(\Delta G_1, \Delta G_2) - c_P(P_1, P_2) = 0.06 \pm 0.015$ .

How to rationalize this difference? A team playing 1:0 gets the same number of points than a team winning 6:0. Whereas in the first case this may have been a fortunate win, in the second case it is very likely that the winning team has been very superior. As a consequence the goal difference may identify very good teams whereas the fitness variation among teams

	$c_P$
$\Delta G$	$0.55 \pm 0.035$
$P$	$0.49 \pm 0.035$
$p(\Delta G) = 2$	$0.19 \pm 0.06$

Table I: Pearson correlation coefficients for different observables.

with a given number of points is somewhat larger. Actually, using  $\Delta G_1$  to predict  $P_2$  is also more efficient than using  $P_1$  ( $c_P(\Delta G_1, P_2) > c_P(P_1, P_2)$ ). One might wonder whether the most informative quantity is a linear combination of  $\Delta G$  and  $P$ . Indeed the optimized observable  $\Delta G + 0.3P$  displays a larger value of  $c_P$  than  $\Delta G$  alone. The difference, however, is so small ( $\Delta c_P \approx 0.001$ ) that the additional information content of the points can be totally neglected.

As a conclusion a final ranking in terms of goals rather than points is preferable if one really wants to identify the strongest or weakest teams.

#### IV. TEMPORAL EVOLUTION OF THE FITNESS

Having identified  $\Delta G$  as an appropriate measure for the team fitness one may ask to which degree the team fitness changes with time. This will be analyzed on three different time scales, now using all data starting from 1965/66.

First we start with variations within a season. One may envisage two extreme scenarios for the time evolution of the fitness during a season: First a random walk in fitness-space, second fluctuations around fixed values. These scenarios are sketched in Fig.3.

To quantify this effect we divide the season in four nearly equal parts (9 matches, 8 matches, 9 matches, 8 matches), denoted quarters. The quarters are enumerated by an index from 1 to 4. In the random-walk picture one would naturally expect that the correlation of quarters 1 and  $m$  ( $m = 2, 3, 4$ ) is the stronger the smaller the value of  $m$  is. For the subsequent analysis we introduce the variable  $n = m - 1$ , indicating the time lag between both quarters. In contrast, in the constant-fitness scenario no dependence on  $n$  is expected. The correlation factors, denoted  $c_q(n)$ , are displayed in the central part of Fig.4. To decrease the statistical error we have averaged over the forward direction (first quarter with  $m = n + 1$ -th quarter) and the time-reversed direction (last quarter with  $m = 4 - n$ -th quarter).



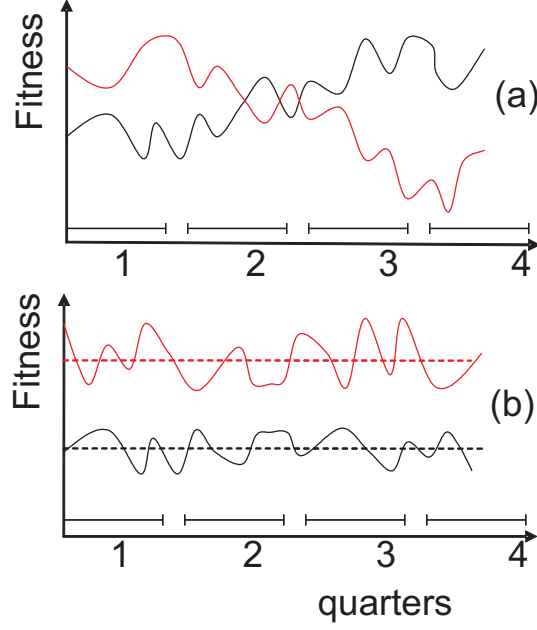


Figure 3: Two extreme scenarios for the time evolution of the fitness during a season. (a) The fitness performs a random-walk dynamics under the only constraint that the fitness distribution of all teams is (roughly) stationary. (b) The fitness of each team fluctuates around a predefined value which is constant for the whole season.

Interestingly, no significant dependence on  $n$  is observed. The correlation between the first and the fourth quarter is even slightly larger than between the first and the second quarter, albeit within the error bars. Thus, the hypothesis that the fitness remains constant during a season (apart from short-ranged fluctuations) is fully consistent with the data. Of course, because of the residual statistical uncertainties of the correlations, one cannot exclude a minor systematic variation of the fitness.

This analysis can be extended to learn about a possible fitness variation when comparing one season with the next or the previous season. More specifically, we correlate the fitness in the first quarter of a given season with the quarters  $m = 5, 6, 7, 8$  in the next season and with the quarters  $m = -3, -2, -1, 0$  and the previous season and plot it again as function of  $n = m - 1$ . The results are also included in Fig.4. Interestingly, there is a significant drop of correlation which, consistent with the previous results, does not change during the course of the next or the previous season. Thus it is by far the summer break rather than the time during a season where most changes happen to the fitness of a team. The very fact that the correlation to last year's result is weaker than present year's result has been

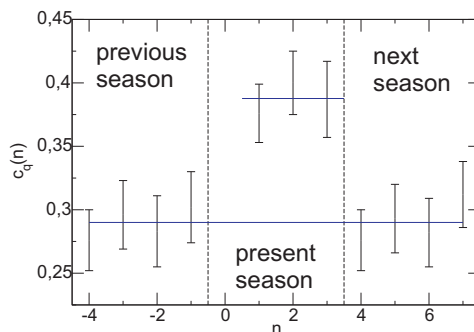


Figure 4: The correlations between quarters, involving the comparison between subsequent seasons.  $n$  denotes the difference between the quarter indices. For a closer description see text.

already discussed in [20], based on a specific model analysis.

Finally, we have analysed the loss of correlation between seasons  $i$  and  $i + n$ . In order to include the case  $n = 0$  in this analysis we compared  $\Delta G$ , determined for the first and the second halves of the season. Thus, for the correlation within the same season one obtains one data point, for the correlation of different seasons one obtains four data points which are subsequently averaged.  $c_y(n)$  denotes the corresponding Pearson correlation coefficient, averaged over all initial years  $i$ . We checked that for  $n > 0$  we get the same shape of  $c_y(n)$  (just with larger values) when full-year correlations are considered. Of course, when calculating the correlation coefficient between seasons  $i$  and  $i + n$  one only takes into account teams which are in the Bundesliga in both years. However, even for large time differences, i.e. large  $n$ , this number is significant (e.g. the number of teams playing in the first season, analyzed in this study, and the season 2007/08 is as large as 11). This already indicates that, given the large number of soccer teams in Germany which might potentially play in the Bundesliga, a significant persistence of the fitness is expected although many of these teams in between may have been briefly relegated to a lower league.

The results are shown in Fig.5.  $c_y(n)$  displays a fast decorrelation for short times which slows down for longer times. To capture these two time-regimes we have fitted the data by a bi-exponential function (numbers are given in the figure caption). This choice is motivated by the fact that this is maybe the simplest function which may quantify the  $n$ -dependence of  $c_y(n)$ . The short-time loss has a time scale of around 2 years. This effect, however, only has an amplitude of around  $2/5$  as compared to the total. The remaining loss of correlation

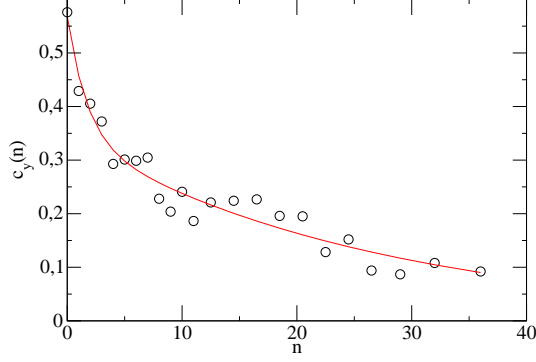


Figure 5: The fitness correlation when comparing  $\Delta G$  for two seasons which are  $n$  years apart. The analysis is based on the comparison of half-seasons (see text for more details). The data are fitted by  $c_y(n) = 0.22 \exp(-n/1.7) + 0.34 \exp(-n/27)$ .

occurs on a much longer scale (around 20-30 years). Obviously, there exist fundamental properties of a team such as the general economic situation which only change on extremely long time scales given the short-range fluctuations of a team composition. As mentioned above, this long-time correlation is also reflected by the small number of teams which during the last decades have played a significant time in the Bundesliga.

## V. STATISTICAL DESCRIPTION OF A SOCCER LEAGUE

### A. General

Here we explicitly make use of the observation that the fitness does not change during the season. Actually, in this Section we will report another supporting piece of evidence for this important fact. Hypothetically, this fitness could be obtained "experimentally" if a season would contain an infinite number of matches between the 18 teams. Then, the fitness could be identified as the observable  $\Delta G(N \rightarrow \infty)$  (abbreviated  $\Delta G(\infty)$ ). The specific value for team  $i$  is denoted  $\Delta G_i(\infty)$ . We already know from the discussion of Fig.2 that the values  $\Delta G_i(\infty)$  are distributed. As a consequence the variance of  $\Delta G(\infty)$ , denoted  $\sigma_{\Delta G}^2$ , is non-zero. Although it cannot be directly obtained from the soccer table (because of the finite length of a season) it can be estimated via appropriate statistical means, as discussed below. Because the number of goals and the width of the distribution of  $\Delta G$  somewhat decreased if comparing the years starting from the season 1987/88 with the earlier years, we

restrict the analysis in this section to the latter time regime.

## B. Estimation of the statistical contribution

Formally, the omnipresence of statistical effects can be written as

$$\Delta G_i(N) = \Delta G_i(\infty) + \Delta G_{i,stat}(N). \quad (2)$$

In physical terms this corresponds to the case of a biased random walk, i.e. a set of particles, each with a distinct velocity (corresponding to  $(\Delta G_i(\infty))$ ) and some diffusion contribution (corresponding to  $\Delta G_{i,stat}(N)$ ). We note in passing that to a good approximation the amplitude of the statistical contribution does not depend on the value of the fitness, i.e. the index  $i$  in the last term of Eq.2 can be omitted. Otherwise, the variance in Fig.2 would depend on the value of  $\Delta G_1$ .

Squaring Eq.2 and averaging over all teams one can write

$$\sigma_{\Delta G(N)}^2 = \sigma_{\Delta G}^2 + \sigma_{\Delta G(N),stat}^2 \quad (3)$$

where the variances of the respective terms have been introduced.  $\sigma_{\Delta G(N),stat}^2$  is expected to scale like  $1/N$  and will disappear in the limit  $N \rightarrow \infty$ . Thus,  $\sigma_{\Delta G}^2$  can be extracted by linear extrapolation of  $\sigma_{\Delta G(N)}^2$  in a  $1/N$ -representation. We have restricted ourselves to even values of  $N$  in order to avoid fluctuations for small  $N$  due to the differences between home and away matches. To improve the statistics we have not only used the first  $N$  matches of a season but used all sets of  $N$  successive matches of a team for the averaging. This just reflects the fact that any  $N$  successive matches have the same information content about the quality of a team.

One can clearly see in Fig.6 that one obtains a straight line in the  $1/N$ -representation for all values of  $N$ . We obtain

$$\sigma_{\Delta G(N)}^2 = 0.215 + \frac{3.03}{N}, \quad (4)$$

i.e.  $\sigma_{\Delta G}^2 = 0.215$  and  $\sigma_{\Delta G(N),stat}^2 = 3.03/N$ . Generally speaking, the excellent linear fit in the  $1/N$ -representation shows again that the team fitness remains stable during the season. Otherwise one would expect a bending because also the first term in Eq.3 would depend on  $N$ ; see again Appendix I for a more quantitative discussion of this effect. Of course, for this

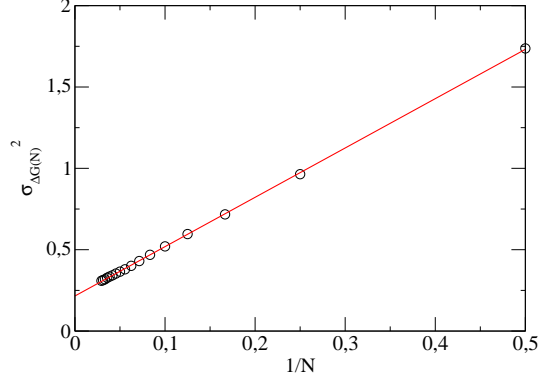


Figure 6: The variance of the distribution of  $\Delta G(N)$ , averaged over all years. The straight line is a linear fit.

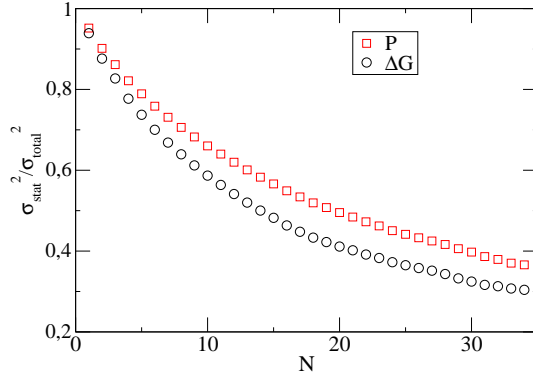


Figure 7: Statistical contribution to the overall variance after  $N$  matches. Included is the analysis for the goal differences as well as for the points.

statement it was important to include only *successive* matches of a team for the statistical analysis.

In Fig.7 the relative contribution of the statistical effects in terms of the variance, i.e.  $\sigma_{\Delta G(N),stat}^2 / (\sigma_{\Delta G(N),stat}^2 + \sigma_{\Delta G}^2)$  is shown as a function of  $N$ . The result implies that, e.g., after the first match of the season ( $N = 1$ ) approx. 95% of the overall variance is determined by the statistical effect. Not surprisingly, the table after one match may be stimulating for the leading team but has basically no relevance for the rest of the season. For  $N \approx 14$  the systematic and the statistical effects are the same. Interestingly, even at the end of the season the statistical contribution in terms of its contribution to the total variance is still as large as 30%.

Repeating the same analysis for the number of points  $P$  yields

$$\sigma_{P(N)}^2 \approx 0.08 + \frac{1.7}{N}. \quad (5)$$

The resulting plot of  $\sigma_{P(N),stat}^2/(\sigma_{P(N),stat}^2 + \sigma_P^2)$  is again displayed in Fig.7. Now it takes even  $N = 22$  matches until the systematic effects start to be dominant. At the end of the season the statistical contribution is as large as 36%. This shows again that  $\Delta G$  is a better measure for the fitness because then the random component in the final ranking is somewhat smaller.

### C. Prediction of team fitness: General framework

The previous analysis has shown that even for  $N = 34$  there still exists a significant random contribution. The next goal is to estimate in a statistically consistent way from knowledge of  $\Delta G(N)$  (e.g. the final scores at the end of the season) the team fitness. Formally, one wants to determine the conditional probability function  $p(\Delta G(\infty)|\Delta G(N))$ . This can be determined by using the Bayes theorem

$$p(\Delta G(\infty)|\Delta G(N)) \propto p(\Delta G(N)|\Delta G(\infty))q(\Delta G(\infty)) \quad (6)$$

Here  $p(\Delta G(N)|\Delta G(\infty))$  is fully determined via Eq.2 and corresponds to a Gaussian with variance  $\sigma_{\Delta G(N),stat}^2$ . The function  $q(\Delta G(\infty))$  describes the a priori probability for the team fitness. This distribution has been already discussed in Fig.1. To first approximation we saw a Gaussian behavior with small but significant deviations. One can show that a strict linear correlation between the estimated fitness (or the behavior in the second half of the season) and  $\Delta G(N)$  is fulfilled for a Gaussian distribution  $q(\Delta G(\infty))$ . Since to a good approximation a linear correlation was indeed observed in Fig.2, for the subsequent analysis we neglect any deviations from a Gaussian by choosing  $q(\Delta G(\infty)) \propto \exp(-\Delta G(\infty)^2/2\sigma_{\Delta G}^2)$ . Of course, for a more refined analysis the non-Gaussian nature, displayed in Fig.1, could be taken into account.

After reordering of the Gaussians in Eq.6 one obtains after a straightforward calculation

$$p(\Delta G(\infty)|\Delta G(N)) \propto \exp[-(\Delta G(\infty) - a_N \Delta G(N))^2/2\sigma_{e,N}^2]. \quad (7)$$

with

$$a_N = \frac{\sigma_{\Delta G}^2}{\sigma_{\Delta G}^2 + \sigma_{\Delta G(N),stat}^2} \quad (8)$$

and

$$\sigma_{e,N}^2 = \frac{\sigma_{\Delta G(N),stat}^2}{1 + \sigma_{\Delta G(N),stat}^2 / \sigma_{\Delta G}^2}. \quad (9)$$

As discussed in the context of Fig.2  $a_N$  is identical to the Pearson correlation coefficient when correlating two subsequent values of  $\Delta G$ , each based on  $N$  matches.

From Eq.4 one obtains  $a_{N=17} = 0.55$  and  $\sigma_{e,N=17}^2 = 0.097$ . As expected  $a_N$  is identical to  $c_P(\Delta G_1, \Delta G_2)$  and within statistical uncertainties identical to the slope of 0.53 in Fig.2.

Finally, we apply these results to the interpretation of the Bundesliga table at the end of the season, i.e. for  $N = 34$ . Using Eq.7 the estimator for  $\Delta G(\infty)$  can be written as

$$\Delta G(\infty) = a_{N=34} \Delta G(N = 34) \pm \sigma_{e,N=34}. \quad (10)$$

For the present data this can be explicitly written as

$$\Delta G(\infty) = 0.71[\Delta G(N = 34) \pm 0.36]. \quad (11)$$

Using standard statistical analysis one can, e.g., determine the probability that a team with a better goal difference  $\Delta G$  (i.e.  $\Delta G_1 > \Delta G_2$ ) is indeed the better team. For the present data it turns out that for  $\Delta G_1 - \Delta G_2 = 0.36$  (corresponding to an absolute value of 12 goals after 34 matches) the probability is approx. 24% that the team with the worse goal difference is nevertheless the better team.

In analogy, one can estimate from Eq.5 that two teams which after the season are 10 points apart have an incorrect order in the league table, based on their true fitness, with a probability of 24%. Maybe this figure more dramatically reflects the strong random component in soccer.

#### D. Prediction of team fitness: Application

These results can be taken to quantify the uncertainty when predicting  $\Delta G_i(M)$  of team  $i$ . More specifically, we assume that this prediction is based on the knowledge of the results of the  $N$  previous matches of team  $i$ . The variance of the estimate of  $\Delta G(M)$  is denoted  $\sigma_{est}^2(M, N)$ . This notation reflects the fact that it depends on both the prediction time scale  $M$  as well as the information time scale  $N$ . To estimate  $\Delta G_i(M)$ , based on  $\Delta G_i(N)$ , two uncertainties have to be taken into account. First, the uncertainty of estimating  $\Delta G_i(\infty)$  is characterized by  $\sigma_{e,N}^2$ . Second, even if  $\Delta G(\infty)$  were known exactly, the statistical uncertainty

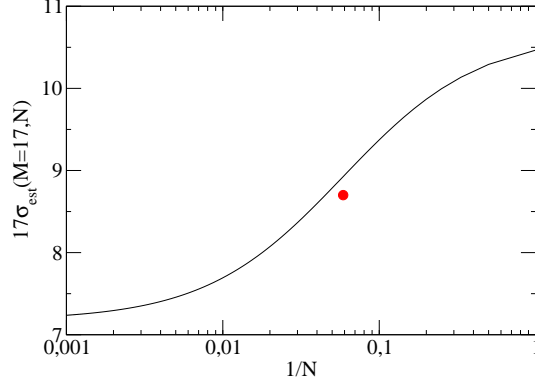


Figure 8: The function  $17\sigma_{est}(M = 17, N)$ , describing the uncertainty for the prediction of the goal difference during the second half of the season based on the knowledge of  $N$  matches. Included is the data point, observed numerically in Fig.2.

of estimating  $\Delta G(M)$  due to the finite  $M$  is still governed by the variance  $\sigma_{\Delta G(M),stat}^2$ . Thus, one obtains

$$\sigma_{est}^2(M, N) = \sigma_{e,N}^2 + \sigma_{\Delta G(M),stat}^2 \quad (12)$$

For the specific choice  $M = 17$ , i.e. for the prediction of the second half of the season, the standard deviation  $17 \cdot \sigma_{est}(M = 17, N)$  of the estimator (expressed in absolute number of goals) is displayed in Fig.8. First, we discuss the extreme cases. In the practically impossible case that the fitness is exactly known (formally corresponding to  $N \rightarrow \infty$ ) one obtains a standard deviation of approx. 7 goals. In the other extreme limit where no information is available, i.e.  $N = 0$ ) one obtains a value of approx. 10.5 goals. Thus the difference between complete information and no information for the prediction of the second half of the season is only 3.5 goals. Finally, for the interpretation of the results in Fig.2 one has to choose  $N = 17$ . As shown in Fig.8 the observed standard deviation of  $17 \cdot 0.51 \approx 8.7$  agrees well with the theoretical value based on Eq.12. The remaining deviations (8.7 vs. 8.9) might reflect the non-Gaussian contributions to  $q(\Delta G(\infty))$ .

From Eq.5 one can estimate in analogy to above that, based on the knowledge of the points for the first half, the number of points for the second half can be estimated with a standard deviation of approx. 6 points. Of course, according to our previous discussion the estimation would be slightly better if the value of  $\Delta G$  rather than the number of points of the first half were taken as input.



	$\langle G_{\pm} \rangle$	$\sigma_{G_+}^2$	$b_{G_+}$	$\sigma_{G_-}^2$	$b_{G_-}$	$c_{+,-}$
Bundesliga	1.43	0.075	1.45	0.055	1.50	0.71
Premier League	1.29	0.075	1.40	0.060	1.40	0.85

Table II: Statistical parameters, characterizing the Bundesliga (1995/96-2007/08) and the English Premier League (1996/97-2006/07).

### E. Going beyond the team fitness $\Delta G$

So far we have characterized the fitness of a team  $\Delta G$ . From a conceptual point of view the most elementary quantities are the number of goals  $G_+$ , scored by a team, as well as the number of goals  $G_-$  conceded by this team ( $\Delta G = G_+ - G_-$ ). Correspondingly,  $\langle G_{\pm} \rangle$  denotes the average number of goals per team and match. The brackets denote the corresponding average. Since the subsequent analysis can be also used for prediction purposes we restrict ourselves to all years since the season 1995/96 when the 3-point rule had been introduced.

The above analysis, performed for  $\Delta G$ , can be repeated for  $G_{\pm}$ . The general notation reads ( $M \in \{G_+, G_-\}$ )

$$\sigma_{M(N)}^2 = \sigma_M^2 + \frac{b_M}{N}. \quad (13)$$

The fitting parameters are listed in Tab.II. We note in passing that all statistical features, described in this Section, are observed in the English Premier League, too. For reasons of comparison the resulting parameters are also included in Tab.II.

For a complete understanding of the goal statistics one has to include possible correlations between  $G_+$  and  $G_-$ , i.e.

$$c_{+,-}(N) = \frac{\langle (G_+ - \langle G \rangle)(\langle G \rangle - G_-) \rangle}{\sigma_{G_+} \sigma_{G_-}}. \quad (14)$$

This value reflects the correlation of a team's strength of attack and defence. Complete correlation means  $c_{+,-}(N) = 1$ . The statistical effects during a soccer match, related to  $G_+$  and  $G_-$ , are likely to be statistically uncorrelated. As a consequence one would not expect a significant  $N$ -dependence. Indeed, we have verified this expectation by explicit calculation of  $c_{+,-}(N)$  which within statistical uncertainty is  $N$ -independent. We obtain  $c_{+,-} = 0.71$ .

This information is sufficient to calculate  $\sigma_{M(N)}^2$  for  $M \in \{\Delta G \equiv G_+ - G_-, \Sigma G \equiv G_+ + G_-\}$  via  $\sigma_{(G_+ \pm G_-)(N)}^2 = \sigma_{G_+(N)}^2 + \sigma_{G_-(N)}^2 \mp 2c_{+,-}\sigma_{G_+}\sigma_{G_-}$ . One obtains  $\sigma_{\Delta G}^2(N) = 0.22 + 2.95/N$

and  $\sigma_{\Sigma G}^2(N) = 0.03 + 2.95/N$ .  $\sigma_{\Delta G}^2(N)$  agrees very well with the data, reported above for the time interval 1987/88-2007/08.

Based on this detailed insight into the statistical nature of goals several basic questions about the nature of soccer can be answered.

Are offence or defence abilities more important? The magnitude of the variance  $\sigma_M^2$  is a direct measure for the relevance of the observable  $M$ . Since  $\sigma_{G_+}^2/\sigma_{G_-}^2 = 1.25 \pm 0.09 > 1$  the investment in good strikers may be slightly more rewarding. However, the difference is quite small so that to first approximation both aspects of a soccer match are of similar importance.

Do teams with good strikers also have a good defence? In case of a strict correlation one would have  $c_{+,-} = 1$ . The present value of 0.71 indicates that there is indeed a strong correlation. However, the residual deviation from unity reflects some team dependent differences beyond simple statistical fluctuations. Interestingly, this correlation is significantly stronger in the Premier League, indicating an even stronger balance between the offence and the defence in a team of the Premier League.

Is the total number of goals of a team (i.e.  $G_+ + G_-$ ) a team-specific property? On average this sum is 97. Without statistical effects due to the finite length of a season the standard deviation of this value would be just  $34\sigma_{\Sigma G} \approx 6$ , i.e. only a few percent. Thus, to a very good approximation the number of goals on average scored by team  $i$  is just given by  $G_{+,i} = \langle G_{\pm} \rangle + \Delta G_i/2$  (an analogous formula holds for  $G_{-,i}$ ).

## VI. SOCCER MYTHS

In typical soccer reports one can read that a team is particularly strong at home (or away) or is just playing a winning streak (*Lauf* in German) or a losing streak. Here we show that the actual data does not support the use of these terminologies (except for the presence of losing streaks).

### A. Home fitness

One may ask the general question whether the overall fitness  $\Delta G$  of the team *fully* determines the *home fitness*, i.e. the team quality of playing at home. If yes, it would be useless

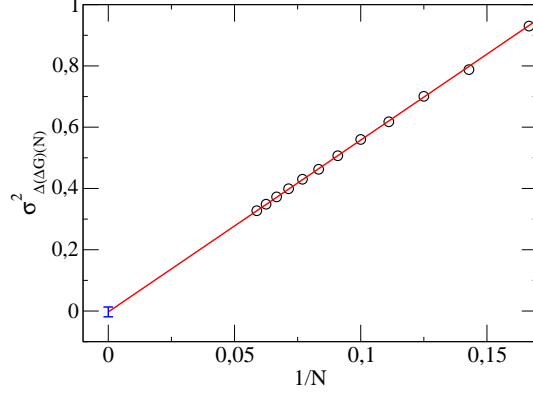


Figure 9: The variance of  $\Delta(\Delta G)$ , i.e.  $\sigma^2_{\Delta(\Delta G)(N)}$  vs.  $1/N$ . The straight line is a linear fit. The extrapolation to  $N = \infty$  yields approx.  $-0.003 \pm 0.016$ .

and misleading to define a team-specific home fitness because it is not an independent observable but just follows from the overall fitness  $\Delta G(\infty)$ . For the present analysis we use again our standard data set starting from 1987/88.

To discuss the ability of a team to play at home as compared to play away we introduce  $\Delta G_H(N)$  and  $\Delta G_A(N)$  as the goal difference in  $N$  home matches and  $N$  away matches, respectively. Of course, one has  $\Delta G_H(N) + \Delta G_A(N) = \Delta G(2N)$ . The *home advantage* can be characterized by

$$\Delta(\Delta G) = \Delta G_H - \Delta G_A. \quad (15)$$

The average value  $\langle \Delta(\Delta G) \rangle$  is approx. 1.4, which denotes the improved home goal difference as compared to the away goal difference. This number also means that on average a team scores 0.7 more goals at home rather than away whereas 0.7 goals more are conceded by this team when playing away. We note in passing that the home advantage is continuously decreasing with time. Just taking the seasons since 1995/96 one gets, e.g.,  $\Delta(\Delta G) \approx 1.0$ .

A team-specific home fitness could be characterized by  $\Delta(\Delta G)_i - \langle \Delta(\Delta G) \rangle$ . A positive value means that team  $i$  is better at home than expected from the overall fitness  $\Delta G$ . Of course, again one has to consider the limit  $N \rightarrow \infty$ . Thus, in analogy to the previous Section one has to perform a scaling analysis. After  $N$  matches  $\Delta(\Delta G)(N)$  will be distributed with a variance, denoted  $\sigma^2_{\Delta(\Delta G)(N)}$ . A positive value of the large  $N$ -limit  $\sigma^2_{\Delta(\Delta G)}$  reflects the presence of a home fitness. Otherwise the quality of a team for a match at home (or away) is fully governed by the overall fitness  $\Delta G(\infty)$ .

The  $N$ -dependence of  $\sigma^2_{\Delta(\Delta G)(N)}$  is shown in Fig.9. To obtain these data one has to

evaluate the appropriate expression for the empirical variance for this type of analysis which is a slightly tedious but straightforward statistical problem. The statistical error has been estimated from performing this analysis for the individual years.

It becomes clear that the hypothesis  $\sigma_{\Delta(\Delta G)(\infty)}^2 = 0$  is fully compatible with the data. Because of the intrinsic statistical error one cannot exclude a finite value of  $\sigma_{\Delta(\Delta G)(\infty)}$  ( $\sigma_{\Delta(\Delta G)(\infty)} < 0.12$ ). This value is less than 10% of the average value  $\langle \Delta(\Delta G) \rangle = 1.4$ . Thus, the presence of teams which are specifically strong at home relative to their overall fitness is, if at all, a very minor effect.

Although this result rules out the presence of a relevant team-specific home fitness it may be illuminating to approach the same problem from a direct analysis of the whole distribution of  $\Delta(\Delta G)(N = 17)$ . The goal is to compare it with the distribution one would expect for the ideal case where no team-specific home fitness is present. This comparison, which is technically a little bit involved, is shifted to Appendix II. It turns out that the residual home fitness can be described by a value of  $0 \leq \sigma_{\Delta(\Delta G)} \ll 0.4$ . This means that in particular the simple model, sketched above, is not compatible with the data. In summary, relative to the average home advantage of 1.4 any possible residual home fitness is a negligible effect.

In literature it is often assumed that for a specific match of team A vs. team B one can a priori define the expectation value of goals  $t_{A(h)}$  and  $t_{B(a)}$ , scored by the home team A and the away team B, respectively. In the approach of Ref.[12] one explicitly assumes  $t_{A(h)} = f_{AB} \cdot c_h$  and  $t_{B(a)} = f_{BA} \cdot c_a$  (using a different notation). Here  $f_{ij}$  contains the information about the offence strength of team  $i$  and the defence strength of team  $j$ . The information about the location of the match is only incorporated into the factors  $c_h$  and  $c_a$ . This approach has two implicit assumptions. First, the fact that  $c_h$  is team-independent is equivalent to the assumption that there is no team-specific home fitness. This is exactly what has been shown in this Section. Second, the average number of goals of, e.g., the home team is proportional to the average number, expected in a neutral stadium. For reasons of convenience this number can be chosen identical to  $f_{AB}$ . Then,  $c_h > 1$  takes into account the general home advantage. The same holds for  $c_a < 1$ . Assuming the multiplicative approach one has to choose

$$c_{h,a} = \frac{\langle G_{\pm} \rangle \pm \langle \Delta(\Delta G) \rangle}{\langle G_{\pm} \rangle}. \quad (16)$$

which for the present case yields  $c_h/c_a \approx 1.45$

In principle, one might have also added some fixed value to take into account the home advantage. Thus, the multiplicative approach is not unique. However, using the above concepts, one can show that this approach is indeed compatible with the data. For this purpose we introduce the observables  $M \in \{G_{+,h}, G_{+,a}, G_{-,h}, G_{-,a}\}$ .  $G_{\pm,h}$  denotes the number of goals scored and conceded by the home team. An analogous definition holds for  $G_{\pm,a}$ . In analogy to above one can calculate  $\sigma_M^2$  obtained again from the  $N \rightarrow \infty$ -extrapolation of the respective observable. One obtains  $\sigma_{G_{+,h}}^2 = 0.089, \sigma_{G_{+,a}}^2 = 0.044, \sigma_{G_{-,h}}^2 = 0.033, \sigma_{G_{-,a}}^2 = 0.069$ . If the properties of home and away goals are fully characterized by the factors  $c_{h,a}$  one would expect  $\sigma_{G_{+,h}}/\sigma_{G_{+,a}} = \sigma_{G_{-,a}}/\sigma_{G_{-,h}} = c_h/c_a$ . The two ratios read 1.4 and 1.45, respectively, and are thus fully compatible with the theoretically expected value of 1.45. In case of an additive constant to account for the home advantage one would have expected a ratio of 1 because then the distributions would have been just shifted to account for the home advantage.

In practical terms this allows one to correct the results of soccer matches for the home advantage by dividing the number of goals in a match by  $c_h$  and  $c_a$ , respectively. This correction procedure may be of interest in cases where one wants to identify statistical properties without being hampered by the residual home advantage. Using this procedure for the data points in Fig.6 the data points for odd  $N$  would also fall on the regression line. We just mention in passing that in the limit of small  $\langle \Delta(\Delta G) \rangle / \langle G_{\pm} \rangle$  and small  $\sigma_{G_{\pm}} / \langle G_{\pm} \rangle$  (which in practice is well fulfilled) this scaling yields similar results as compared to a simple downward shifting of the home goals and upward shifting of the away goals by  $\langle \Delta(\Delta G) \rangle / 2$ .

## B. Streaks

The aspect of identifying winning or losing streaks is somewhat subtle because one has to take care that no trivial selection effects enter this analysis. Here is one example of such an effect. Evidently, in case of a winning streak it is likely that during this period the team played against somewhat weaker teams and will, subsequently, on average play against somewhat stronger teams. Thus, to judge the future behavior of this team one needs a method which takes these effects in a most simple way into account. To obtain a sufficiently good statistics here we use our complete data set, starting from the season 1965/66.

The key question to be answered here is whether or not the presence of a winning or



Figure 10: Sketch of the definitions of  $n$  and  $m$  for the analysis of the possible existence of winning and losing streaks.

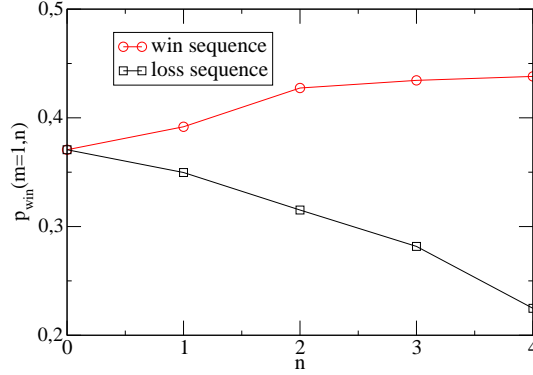


Figure 11: The probability  $p_{win}(n, m)$ . to win after a team as won or lost  $n$  times.

losing sequence stabilizes or destabilizes a team or maybe has no effect at all. If a winning sequence stabilizes a team one may speak of a winning streak. Analogously, if a losing sequence destabilizes a team one has a losing streak. In general, we have identified all sequences of  $n$  successive matches where  $n$  wins or losses were present. Of course, the actual length of the win or loss sequences can have been much longer. Having identified such a sequence we have determined the probability that in the  $m$ -th match after this sequence that team will win. This probability is denoted  $p_{win}(m, n)$ . This is sketched in Fig.10 for the case  $n = 4$ .

In a first step we analyze the winning probability in the next match, i.e. for  $m = 1$ . The data are shown in Fig.11. In case of winning sequence the probability to win increases with increasing  $n$ . The opposite holds for a losing sequence. Does this indicate that the longer the winning (losing) sequence, the stronger the (de)stabilization effect, i.e. real winning or losing streaks emerge?

This question has been already discussed in Ref. [14]. It was correctly argued that by choosing teams which have, e.g., won 4 times one typically selects a team with a high fitness. This team will, of course, win with a higher probability than an average team (selected for  $n = 0$ ). Thus the increase of the win probability with  $n$  is expected even if no stabilizing

effect is present. It would be just a consequence of the presence of the fitness distribution and thus of good and bad teams, as shown above. Only if all teams had the same fitness the data of Fig.11 would directly indicate the presence of a stabilization and destabilization effect, respectively.

The key problem in this analysis is that the different data points in Fig.11 belong to different subensembles of teams and thus cannot be compared. Therefore one needs to devise an analysis tool, where a fixed subensemble is taken. The realization of this tool is inspired by 4D NMR experiments, performed in the 90s in different groups to unravel the properties of supercooled liquids [16, 17, 18]. The key problem was to monitor the time evolution of the properties of a specific subensemble until it behaves again like the average. This problem is analogous to that of a soccer team being selected because of  $n$  wins or losses in a row.

This idea can be directly applied to the present problem by analyzing the  $m$ -dependence of  $p_{win}(m, n)$ . It directly reflects possible stabilization or destabilization effects. In case of a stabilization effect  $p_{win}(m)$  would be largest for  $m = 1$  and then decay to some limiting value which would be related to the typical fitness of that team after possible effects of the series have disappeared. In contrast, in case of a destabilization effect  $p_{win}(m = 1)$  would be smaller than the limiting value reached for large  $m$ . Note that in this way the problem of different subensembles is avoided. Furthermore this analysis is not hampered by the fact that most likely the opponents during the selection period of  $n$  matches were on average somewhat weaker teams. The limiting value has been determined independently by averaging  $p_{win}(m, n)$  for  $|m| > 8$ , i.e. over matches far away from the original sequence. To improve the statistical quality this average also includes the matches sufficiently far before the selected sequence (formally corresponding to negative  $m$ ). Of course, only matches within the same season were taken into account. It is supposed to reflect the general fitness of a team during this season (now in terms of wins) independent of that sequence. In case of no stabilization or destabilization effect the observable  $p_{win}(m, n)$  would not depend on  $m$ . This would be the result if playing soccer would be just coin tossing without memory. To avoid any bias with respect to home or away matches we only considered those sequences where half of the matches were home matches and the other half away matches ( $n$  even). Furthermore, the data for  $p_{win}(m, n)$  are averaged pairwise for subsequent  $m$  (1 and 2, 3 and 4, and so on).

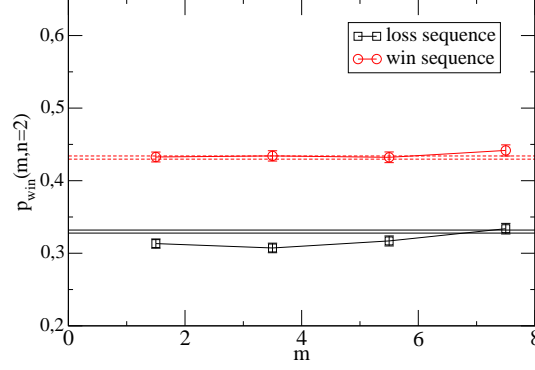


Figure 12: The probability to win  $p_{win}(m, n = 2)$  after a sequence of  $n = 2$  wins and losses, respectively. The broken lines indicate the range ( $\pm 1\sigma$ -interval) of the plateau value reached for large  $m$ .

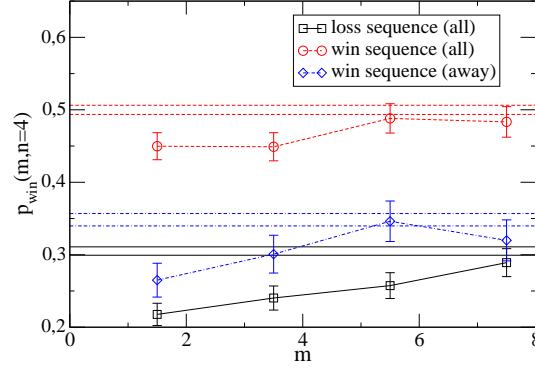


Figure 13: Same as in the previous figure for  $n = 4$ . In addition we have included data where only away matches of the teams are considered for the calculation of  $p_{win}(m, n = 4)$  in case of a win sequence.

The functions  $p_{win}(m, n)$  for  $n = 2$  and  $n = 4$  are shown in Figs. 12 and 13, respectively. For  $n = 4$  a total of 374 win sequences and 384 loss sequences have been taken into account. For  $n = 2$  one observes a small but significant destabilization after a loss sequence. It takes approx. 8 matches to recover. No effects are seen for the win sequence. More significant effects are visible for  $n = 4$ . For the loss sequence one observes that directly after the selected sequences, i.e. for  $m = 1$  and  $m = 2$  the winning probability is reduced by approx. 30% as compared to the limiting value. Thus for about 6 matches the teams play worse than normal. Surprisingly, a reduction of  $p_{win}(m, n = 4)$  for small  $m$  is also visible for the win sequence. Thus, there seems to be a destabilization rather than a stabilization effect. By restricting the analysis to the away matches after the selected sequence, this effect is



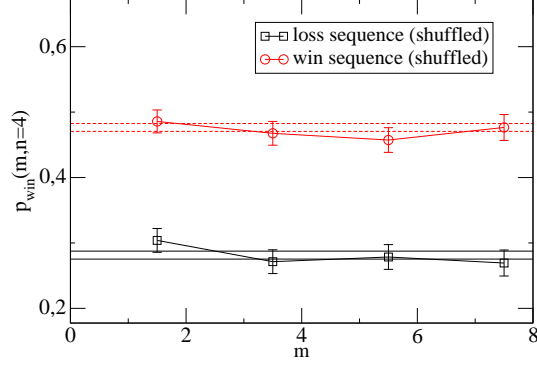


Figure 14: Analysis of loss and win sequences, using shuffled data.

even more pronounced. Of course, correspondingly the effect is smaller for home matches. Unfortunately,  $n = 6$  can no longer be analyzed because due to the small number of events the statistics is too bad.

Of course, a critical aspect in this discussion is the matter of statistical significance. For this purpose we have estimated the probability that, using Gaussian statistics, the average of the first four matches after a win sequence can be understood as an extreme statistical deviation from the final plateau value. This probability turns out to be smaller than  $10^{-3}$ . Furthermore we analyzed shuffled data, i.e. where for a given team in a given season the 34 matches are randomly ordered. The results for  $p_{win}(m, n = 4)$ , using one example of ordering, are shown in Fig.14. As expected no effect is seen. The observation that the plateau values are somewhat lower than in Fig.13 just reflects the fact the the first data points (small  $m$ ) in Fig.13 are systematically lower than the respective plateau value.

Thus, we conclude that both a positive ( $n = 4$ ) as well as a negative sequence ( $n = 2, 4$ ) have a destabilizing effect. This means that losing streaks indeed exist whereas there are no stabilization effects for positive sequences, invalidating the notion of a winning streak. Rather destabilization effects occur after a longer winning sequence. This asymmetry between positive and negative sequences is already reflected by the asymmetry, seen in Fig.11.

Actually, the present results disagree with the statistical analysis in Ref.[21] for the Premier League. In that work it is concluded that sequences of consecutive results tend to end sooner than they should without statistical association. However, the presence of losing streaks has been clearly demonstrated above. The disagreement might be due to the different data set (Bundesliga vs. Premier League). However, one needs to take into account that in that work the results have been obtained within a framework of a specific

model via Monte-Carlo simulations. The present analysis has the advantage that, first, it does not refer to any model about the nature of soccer and, second, can be done without additional Monte-Carlo simulations. Thus, possible artifacts of the model might hamper the interpretation of the data.

## VII. DISCUSSION AND SUMMARY

On a conceptual level we have used finite-size scaling methods to extract the underlying distribution of fitness parameters. It turns out that the goal difference is a better measure of the team fitness than the number of points. From a technical point of view a key aspect was to analyze the  $N$ -dependence of observables such as  $\Delta G$ . This problem is analogous to the simple physical problem of random walks with a drift. The key results can be summarized as follows.

1.) The fitness of a team displays a complex temporal evolution. Within a season there are no indications for any variations (except maybe for day-to-day fluctuations around some average team fitness which can only be identified via a single-match analysis. This is, however, beyond the scope of the present work). During the summer-break a significant decorrelation is observed. This short-scale decorrelation stops after around 2 years where approx. 40% of the fitness has been changed (some teams becoming better, some worse). Interestingly, the remaining 60% of the fitness only decorrelates on an extremely long timescale of 20-30 years which is close to the data window of our analysis. This shows that there are dramatic persistence effects, i.e. there are some underlying reasons why good teams remain good on time scales largely exceeding the lifetime of typical structures in a club (manager, coach, players etc.).

2.) For finite seasons (which, naturally, is realized in the actual soccer leagues) the fitness of a team can be only roughly estimated because of the presence of residual statistical fluctuations. However, by linear extrapolation of the variance of the team fitness one can identify the underlying variance one would (hypothetically) obtain for an infinite number of matches. Based on this one can estimate the statistical contribution to the end-of-the-season table which is quite significant (36% for points). This allows one to quantify, e.g., the relevance of the final league table in some detail.

3.) The overall fitness, defined via the goal difference  $\Delta G$ , is to a large extent the only

characteristics of a team. In particular there is no signature of the presence of a team specific home fitness. We would like to stress that the definition of a home fitness is always relative to a single season. This means if a team is strong at home in one year and weak in another year this would nevertheless show up in the present analysis. Whenever a team plays better or worse at home than expected (measured via  $\Delta G_H - \Delta G_A$ ) this effect can be fully explained in terms of the natural statistical fluctuations, inherent in soccer matches.

4.) A more detailed view on the number of goals reveals that the quality of the offence and that of the defence of a team is strongly correlated. In case of a perfect correlation their quality would be fully determined by the overall fitness. However, since the correlation is not perfect there exist indeed differences. Furthermore, the strength of attack is slightly more important for a successful soccer team than the strength of defence, albeit the difference is not big.

5.) It is possible to identify the impact of the home-advantage for the final result. Stated differently, one can estimate the average outcome of a match one would obtain at a neutral stadium. This procedure may be helpful if data are taken as input for a statistical analysis.

6.) The notion of streaks, as present in the soccer language, can only be confirmed in case of a losing streak. This means that if a team has lost several times (we analyzed 2 and 4 times) there is a significant drop of their fitness as compared to the normal level which will be reached again sufficiently far away from the time period. Possible reasons may be related to psychological aspects as well as the presence of persistent structural problems (such as heavily injured players). Surprisingly, no winning streak could be identified. Winning two times had no effect on the future outcome. Winning four times even reduced the fitness, in particular when having an away match. This analysis had to be performed with care in order to avoid any trivial statistical effects. Possibly, this indicates an interesting psychological effect. In literature one can find models for understanding the basis of human motivation. In one of the standard models by Atkinson a reduction of motivation may occur if the next problem *appears* either to be too difficult (after having lost several times) or too simple (after having won several times) [22]. However, since these types of sequences (for  $n = 4$ ) of wins or losses are relatively rare they are of very minor relevance for the overall statistical description of the temporal evolution of soccer matches. Since furthermore the effect of sequences decays after a few more matches (up to 8) these observations are consistent with the notion that the fitness does not change during a season (if averaged over the time scale

of at least a quarter season).

Of course, a further improvement of the statistical analysis could be reached if further explanatory variables are implemented such as the possession of the ball [23]. It would be interesting to quantify the increase of the predictive power in analogy to the analysis of this work; see, e.g., Tab.I.

Whereas some of our results were expected, we had to revise some of our own intuitive views on how professional soccer works. Using objective statistical methods and appropriate concepts, mostly taken from typical physics applications, a view beyond the common knowledge became possible. Probably, for a typical soccer fan also this statistical analysis will not change the belief that, e.g., his/her support will give the team the necessary impetus to the next goal and finally to a specific home fitness. Thus, there may exist a natural, maybe even fortunate, tendency to ignore some objective facts about professional soccer. We hope, however, that the present analysis may be of relevance to those who like to see the systematic patterns behind a sports like soccer. Naturally, all concepts discussed in this work can be extended to different types of sports. Furthermore an extension to single-match properties as well as a correlation with economic factors is planned for the future.

We would like to thank S.F. Hopp, C. Müller and W. Krawtschunowski for the help in the initial phase of this project as well as B. Strauss, M. Tolan, M. Trede and G. Schewe for interesting and helpful discussions. Furthermore we would like to thank H. Heuer for bringing the work of Atkinson to our attention.

## VIII. APPENDIX I

Here we consider a simple model which further rationalizes the statement that observables with larger Pearson correlation coefficients (correlation between first and second half of season) are better measures for the fitness of a team. This holds independent of whether or not the true fitness changes during a season or remains constant. We assume that the true fitness of a team  $i$  at time  $j$  ( $j$  may either reflect a single match or, e.g., the average fitness during the  $j$ -th half of the season) can be captured by a single number  $\mu_{i,j}$ . Evidently, the true fitness  $\mu_{i,j}$  of team  $i$  is not exactly known. The variance of the fitness  $\sigma_\mu^2$  is assumed to be time independent, which just reflects stationarity.

In the experiment (here: soccer match) one observes the outcome  $x_{i,j}$  which may, e.g.,

correspond to the goal difference or the number of points of team  $i$  at time  $j$ . We assume a Markovian process, i.e. the outcome at time  $j$  is not influenced by the outcome in previous matches. Naturally  $x_{i,j}$  is positively correlated with  $\mu_{i,j}$ . Without loss of generality we assume that the  $\langle \mu_{i,j} \rangle_i = \langle x_{i,j} \rangle_i = 0$ . The index  $i$  reflects the fact that the averaging is over all teams. For reasons of simplicity we assume a linear relation between  $x_{i,j}$  and  $\mu_{i,j}$ , namely

$$x_{i,j} = a(\mu_{i,j} + \xi). \quad (17)$$

Here  $a > 0$  is a fixed real number and  $\xi$  some noise, characterized by its variance  $\sigma_\xi^2$ . The noise reflects the fact that the outcome of a soccer match is not fully determined by the fitness of the teams but also includes random elements. This relation expresses the fact that a team with a better fitness will on average also perform better during its matches.

The key idea in the present context is to use the outcome of matches to *estimate* the team fitness. The degree of correlation between  $x_{i,j}$  and  $\mu_{i,j}$  is captured by the correlation coefficient

$$c_{x_j, \mu_j} = \frac{\langle x_{i,j} \mu_{i,j} \rangle_i}{\sigma_x \sigma_\mu}. \quad (18)$$

A large value of  $c_{x_j, \mu_j}$  implies that the estimation of  $\mu_{i,j}$ , based on knowledge of  $x_{i,j}$  works quite well. Thus, one may want to search for observables  $x_{i,j}$  with large values of  $c_{x_j, \mu_j}$ . Unfortunately, since  $\mu_{i,j}$  cannot be measured  $c_{x_j, \mu_j}$  is not directly accessible from the experiment. The theoretical expectation reads (see Eqs. 17 and 18)

$$c_{x_j, \mu_j} = \frac{\sigma_\mu}{\sqrt{\sigma_\mu^2 + \sigma_\xi^2}}. \quad (19)$$

For a closer relation to the general experimental situation one has to take into account that the team fitness may somewhat change with time. This can be generally captured by the correlation factor

$$c_{\mu_j, \mu_{j+1}} = \frac{\langle \mu_{i,j+1} \mu_{i,j} \rangle}{\sigma_\mu^2}. \quad (20)$$

Experimentally accessible is the correlation of  $x_{i,j}$  for two subsequent time points  $j$  and  $j+1$ . A short and straightforward calculation yields (using Eq.19)

$$c_{x_j, x_{j+1}} = c_{\mu_j, \mu_{j+1}} [c_{x_j, \mu_j}]^2. \quad (21)$$

This result shows that *independent* of the possible decorrelation of the true fitness  $\mu$  observables  $x$  with a larger correlation coefficient  $c_{x_j, x_{j+1}}$  display larger  $c_{x_j, \mu_j}$ , i.e. form a better

measure for the true fitness  $\mu$ . This is the line of reasoning used to identify  $\Delta G$  as a better fitness measure than the number of points independent of whether or not  $\Delta G$  changes during a season.

To go beyond this key statement we specify the loss of correlation of the true fitness via the simple linear ansatz

$$\mu_{i,j+1} = b\mu_{i,j} + \epsilon. \quad (22)$$

Here the noise term is characterized by the variance  $\sigma_\epsilon^2$ . For reasons of simplicity we assume that the random-walk type dynamics is identical for all teams. Stationarity is guaranteed exactly if

$$\sigma_\epsilon^2 = \sigma_\mu^2(1 - b^2). \quad (23)$$

Constant fitness naturally corresponds to  $b = 1$  and  $\sigma_\epsilon = 0$ . Of particular interest for the present work is the average of  $x_{i,j}$  over  $N$  times (e.g.  $N$  matches if  $j$  counts the matches). Here we define

$$X_i(N) = \frac{\sum_{j=1}^N x_{i,j}}{N}. \quad (24)$$

The variance of this average, denoted  $\sigma_{X(N)}$  can be calculated in a straightforward manner. The result reads

$$\sigma_{X(N)}^2 = \frac{a^2\sigma_\mu^2}{N^2} \left[ N + 2b \frac{N-1-Nb+b^N}{(1-b)^2} \right] + \frac{a^2\sigma_\xi^2}{N}. \quad (25)$$

For  $b = 1$  one obtains  $\sigma_{X(N)}^2 = a^2\sigma_\mu^2 + a^2\sigma_\xi^2/N$ . Thus, in case of constant team fitness one gets a linear behavior in the  $1/N$  representation and the limit value just corresponds to the variance of the team fitness (apart from the trivial constant  $a$ ). This implies that by extrapolation one can get important information about the underlying statistics, as described by the true team fitness  $\mu_{i,j}$ . This just reflects the fact that for sufficient averaging the noise effects become irrelevant. For  $b < 1$ , however, one has a crossover from that behavior to  $\sigma_{X(N)}^2 = a^2\sigma_\mu^2[(1+b)/(1-b)]/N + a^2\sigma_\xi^2/N$  for large  $N$ , thus approaching zero for large  $N$ . Since  $\sigma_{\Delta G}^2(N)$  did not show any bending we have concluded in the main text that the data do not indicate a decorrelation of the fitness within a single season.

## IX. APPENDIX II

Here we discuss in more detail the distribution of  $\Delta(\Delta G)(N = 17)$  shown in Fig.15. Of course, it has a finite width due to statistical effects. Our goal is to compare this distribution

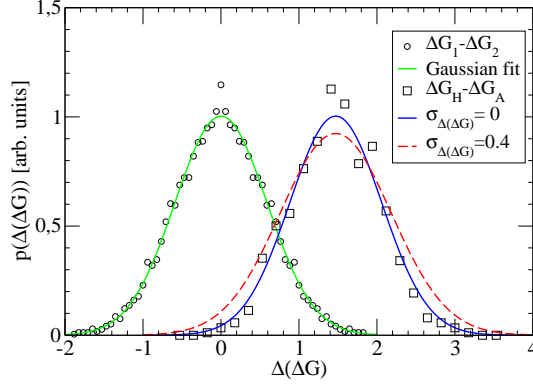


Figure 15: Analysis of the home fitness. The squares correspond to the actual distribution of  $\Delta(\Delta G)$ . This curve is compared with the estimation for  $\sigma_{\Delta(\Delta G)} = 0$  and  $\sigma_{\Delta(\Delta G)} = 0.4$ . For more details see text.

with a second distribution which is generated under the assumption that no specific home fitness exists. For this purpose we have defined, for each team in a given season, the random variable  $\Delta G_1 - \Delta G_2$ . Here the first term contains the average of the goal differences of some 17 matches and the second term the average over the remaining 17 matches. The 34 matches were attributed to both terms such that the number of home matches of the first term is 9 (or 8) and that of the second team is 8 (or 9), respectively. Then we have generated the distribution of  $\Delta G_1 - \Delta G_2$ . In order to get rid of the residual home effect (9 vs. 8) we have shifted this curve so that the average value is 0. This procedure has been repeated for many different mappings of this kind and for all seasons. The resulting curve is also shown in Fig.15. It reflects the statistical width of  $\Delta(\Delta G)$  after a season if no home advantage were present. It can be very well described by a Gaussian. When shifting this distribution by the value of the average home advantage one obtains an estimate of the distribution of  $\Delta(\Delta G)$  for  $\sigma_{\Delta(\Delta G)}^2 = 0$ . To be consistent with this procedure we have generated the distribution of  $\Delta(\Delta G)$  ( $N = 17$ ) in an analogous way. We have calculated this distribution for every individual season and shifted each curve so that the mean agrees with the overall mean. In this way we have removed a possible broadening of this curve due to the year-to-year fluctuations of the general home advantage.

In agreement with the discussion of Fig.9 one observes a good agreement with the actual distribution of  $\Delta(\Delta G)$ . By convolution of this distribution with a Gaussian with variance  $\sigma_{\Delta(\Delta G)}^2$  one can get information about the sensitivity of this analysis. Choosing, e.g.,

$\sigma_{\Delta(\Delta G)} = 0.4$ , one can clearly see that this choice is not compatible with the actual distribution of  $\Delta(\Delta G)$ . Thus, if at all, the residual home fitness can be described by a value of  $\sigma_{\Delta(\Delta G)}$  significantly smaller than 0.4. In the main text we have derived an upper limit of 0.12.

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